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Robust Fault Detection of Jet Engine Sensor Systems Using Eigenstructure Assignment

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This paper examines a robust fault detection scheme that can be used to detect faulty sensors of jet engines. The fault detection scheme has to be insensitive to disturbances while being highly sensitive to sensor faults (robust). The paper presents a complete description of a robust fault detection approach based on eigenstructure assignment, both in continuous- and discrete-time domains. By assigning the left (or right) eigenvectors of the observer orthogonal (or parallel) to the disturbance directions, the robust (disturbance decoupling) fault detection is achieved. The approach has been applied to a realistic jet engine simulation system. The system is a 17th-order system, and a reduced-order model is used to approximate the system. Modeling errors are considered as disturbances acting on the fault detection scheme. A particularly novel feature of the work is the development and use of a new method (new in this context) for estimating disturbance direction. The robust fault detection scheme design uses this estimated direction as that of the direction of unknown inputs (disturbances). Simulation results show that the scheme can detect soft or incipient faults efficiently.

I. Introduction

IN recent years hydromechanical implementations of turbine engine control systems have matured into highly reliable units.[1,2] Engines and control systems have become more complex to meet ever-increasing performance requirements. The need for sensor fault detection and isolation (FDI) for jet engines has arisen as advanced digital controllers have been introduced to improve fuel economy and control accuracy. To improve the overall reliability of the engine system, various redundancy management techniques have to be applied to both the total control system and individual components. Sensors are the least reliable of the control system components, and, hence, some form of sensor redundancy is necessary to achieve adequate closed-loop reliability.

There are two types of sensor redundancy: hardware and analytical. *Hardware redundancy* uses multiple sensors to measure the same engine variables. Typically, a voting scheme is used to detect and isolate faults. *Analytical redundancy* uses a reference model of the engine and redundant information in dissimilar sensors to provide an estimate of a measured variable. The difference between estimates and measurements can be used to declare faults. Analytical redundancy makes use of a mathematical model of the engine and is, therefore, often referred to as the *model-based approach*.[3] The movement toward computer controllers has made the inclusion of sensor FDI techniques based on analytical methods possible. As far as sensor FDI is concerned, turbofan engines can be characterized in a number of ways. These include:

1) The high penalty for complete engine malfunction requires that all system components must have a very high individual reliability; movement toward digital controllers has made possible the inclusion of sensor FDI techniques based on analytical redundancy.

2) The sensors are working in a harsh environment, so their fault probabilities are comparable with the overall system fault requirements. This necessitates that some form of FDI and subsequent fault accommodation be used.

3) The available computing power is limited, as it is relatively expensive to ensure the high reliability of electronic systems in the harsh environment.

4) The bulk, weight, and costs imposed by the sensors are high; hence, "efficiency" in the sensor set design is required.

5) A large number of different measurements are required, both for control and for the prompt detection of processes exceeding safe limits (fault).

Hardware redundancy results in more costly, heavier, less practical, and less potentially reliable systems than do various analytical redundancy strategies. Because cost, weight, and reliability are important issues in turbine engine control systems design, much research interest has been focused on analytical redundancy strategies. Significant attention has been paid in the literature to the FDI theory based on analytical (functional) redundancy. The relatively recent survey papers[1-3] provide a very thorough review of previous and current research programs in this subject. The most comprehensive practical feasibility study to date is the NASA Lewis research program, first reported by Beattie et al.[4] This study discusses a wide variety of FDI schemes and selected a Kalman filter with generalized likelihood ratio (GLR) testing-based scheme as a candidate for further development. A paper by Leininger[5] examines the impact of an inaccurate model on Kalman filter-based fault detection procedures. The paper demonstrates that model inaccuracies appear as biases in the residuals. This indicates that the Kalman filter-based approach does not have an associated "robust design" strategy and is not really classified as having the best potential for robust performance. Later the whole scheme was rig tested.[6,7] These studies have shown that the theory of sensor FDI can be used in practical turbofan sensors FDI. The gas turbine engine is a very nonlinear dynamic system; as the system is so complex, there is no one complete nonlinear model, and the engine is considered as an uncertain system. The *modeling uncertainty* presents a challenge to fault detection design to meet robustness requirements. Additionally, there has been further work[8] arising from the original NASA contract that addresses the uncertainties of the system.

Certainly, the reliability of an FDI scheme must be higher than the monitored system. The main problem obstructing the progress and improvement in reliability of FDI schemes is the *robustness problem with respect to uncertainty*, which arises, for example, due to process noise, turbulence, parameter variations, and modeling error. All analytical redundancy approaches employ a model of the monitored system, and, if no uncertainties exist on the system (i.e., the model is accurate

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and the characteristics of all disturbances are known), FDI is very straightforward and reliable detection is easily achievable. In most systems, however, uncertainties are present almost inevitably and may interfere seriously with the FDI procedure.[3] A practical FDI scheme must be robust. The jet engine system is a highly nonlinear system and offers challenges to the design of a reliable and robust monitoring system. The noise also has an influence on FDI, but in this article we are only concerned with the robustness problem with respect to uncertainties, as the effect of noise on FDI is well understood. There are several effective techniques for FDI in the presence of noise. They all use window averaging to reduce the contribution of noise to the detection function while not reducing the fault signature. These techniques include the sequential probability ratio test (SPRT), the GLR test, and the weighted sum square residual (WSSR) test; the details can be seen in Refs. 3, 9, and 10.

References 11-17 have studied the effect of uncertainties on FDI, and some robust design measures have been taken. However, the robust fault detection is still an open problem. The term *robustness* can be associated with a fault detection scheme. As a general definition, the robustness is the degree to which the fault detection performance is unaffected by (or remains insensitive to) uncertainties of the system. As the residual is employed as a declarative signal in FDI, the robustness can be measured by using the sensitivities of the residual to uncertainties and faults. A robust fault detection scheme is such a scheme, whose residual is insensitive to uncertainties and sensitive to faults. The aim of robust design of the fault detection scheme is to reduce the effects of uncertainties on the residuals or to enhance the effects of faults acting on the residuals.

An important kind of robust technique for FDI is the "disturbances (unknown inputs) decoupling" approach. In this approach, all uncertainties are summarized as disturbances (unknown inputs) acting on the system model. Based on this assumption, one can use the "unknown input observer" to estimate the state. Watanabe and Himmelblau,[11] Wünnenberg and Frank,[12] Frank and Wünnenberg,[13] and Wünnenberg[14] have used the unknown input observer approach to the robust FDI. In this scheme, the unknown input (disturbance) does not affect the state estimation error, so that robust detection is achievable.

Patton et al.[15-17] have shown that the same design goal can be achieved by using eigenstructure assignment. By assigning the eigenstructure of the observer, the residuals can be decoupled from the disturbances, thus enabling robust detection of faults to be achieved. Note that White and Speyer[18] have also used eigenstructure assignment technique to fault (failure) detection filter design, but this is a different problem with robust FDI. This paper presents the complete description of the robust fault detection method based on the eigenstructure assignment, both in continuous-time and discrete-time domains. The paper focuses attention on a real jet engine sensor system. Attention has also been paid to the estimation of the disturbance distribution matrix E. Accompanying all the disturbance decoupling approaches, an important assumption is that the disturbance distribution matrix be known, but within the framework of international research on this subject a generalized approach for obtaining this matrix is still lacking. This paper proposes a method for estimating the columns of the matrix E by using the augmented system method. Once this matrix is known, albeit estimated, a robust fault detection scheme can be designed. The fault monitor scheme is assessed by using the simulation corresponding to different operating points. The results show that nearly perfect performance is achievable, even under different plant operation.

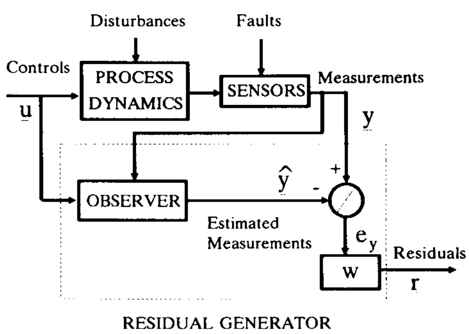

Fig. 1 The block diagram of an observer-based residual generator.

II. Robust Fault Detection Using Eigenstructure Assignment

A. Problem Specification

To approach the problem from the most general point of view, one must start with a mathematical system description that includes all kinds of dynamic input signals that can occur in practice and affect the dynamic behavior of the system. The state-space model of the system is therefore given by

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) + f(t) \tag{2}$$

where $x(t)$ represents the $n \times 1$ state vector, $u(t)$ the $r \times 1$ known input vector with the corresponding input distribution matrix B, and $y(t)$ the $m \times 1$ measurement vector. The term $Ed(t)$ characterizes a $q \times 1$ unknown input (disturbance) vector $d(t)$ with known distribution matrix E acting directly on the system dynamics equation. Hence, $Ed(t)$ is used to represent all uncertainties (if possible) acting on the system model. The faults that corrupt the measurements, called sensor faults, are described by the vector $f(t)$.

Model-based fault diagnosis can be considered as two stages: *residual generation* and *decision making*.[1-3,9,10] In the first stage, outputs and inputs of the system are processed by an appropriate algorithm (a processor) to generate residual signals, which are quantities that represent the inconsistency between the actual plant measurements and the mathematical model outputs. In the second (decision making) stage, the residuals are examined for the likelihood of faults, and a decision rule is then applied to determine if any fault has occurred. The residuals are usually based on a (weighted) comparison between the actual outputs and the estimated outputs generated from the mathematical model. The principle of the observer-based approach for generating of robust residuals is illustrated in Fig. 1. The residual generator uses an observer, which generates estimates of the system states and outputs, and provides residual signals, which are independent of uncertainties. The observer dynamics are described by the following:

$$\dot{\hat{x}}(t) = (A - KC)\hat{x}(t) + (B - KD)u(t) + Ky(t) \tag{3}$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t) \tag{4}$$

where $\hat{x}(t)$ is the $n \times 1$ state estimation vector and $\hat{y}(t)$ the $m \times 1$ output estimation vector. The state estimation error $[e(t) = x(t) - \hat{x}(t)]$ dynamics are as follows:

$$\dot{e}(t) = A_c e(t) + Ed(t) - Kf(t) \tag{5}$$

where $A_c = A - KC$. A $p \times 1$ residual vector is defined as

$$r(t) = We_y(t) = W\left[y(t) - \hat{y}(t)\right] \tag{6}$$

where W is a $p \times m$ weighting matrix.

$$r(t) = WCe(t) + Wf(t) = He(t) + Wf(t) \tag{7}$$

where $H = WC$ is a $p \times n$ matrix. From Eqs. (5-7), the complete response of the residual vector is

$$r(s) = \left[W - WC(sI - A_c)^{-1}K\right] f(s) + WC(sI - A_c)^{-1} Ed(s) \tag{8}$$

One can see that the residual is not zero, even if no faults occur in the system. Indeed, it can be difficult to distinguish the effects of faults from the effects of disturbances. The effects of disturbances (unknown inputs) obscure the performance of FDI and act as a source of false alarms. Therefore, to minimize the false alarm rate, one should design the observer such that its residual becomes decoupled from disturbances.

B. Eigenstructure Assignment Approach for Robust Fault Detection

To make the residual $r(t)$ independent of uncertainties, it is necessary to null the entries in the transfer function matrix between disturbances and the residual, i.e.,

$$G_{rd}(s) = WC\left[sI - (A - KC)\right]^{-1} E = 0 \tag{9}$$

This is a special case of the *output-zeroing* problem, which is well known in multivariable control theory.[19] The remaining problem is to choose the matrices K and W to satisfy Eq. (9), in addition to choosing the suitable eigenvalues to optimize the FDI performance. The eigenstructure assignment is a direct way to solve this problem. First, we quote two lemmas to show the relationship between the eigenvectors and eigenvalues (the eigenstructure) of a system.

Lemma 1: A given left eigenvector l_i^T (corresponding to eigenvalue β_i) of A_c is always orthogonal to the right eigenvectors v_j, corresponding to the remaining $(n-1)$ eigenvalues β_j of A_c, where $\beta_i \neq \beta_j$.

Lemma 2: Any transfer function matrix can be expanded in term of eigenstructure:

$$(sI - Ac)^{-1} = \frac{v_1 l_1^T}{s - \beta_1} + \cdots + \frac{v_n l_n^T}{s - \beta_n} \tag{10}$$

where v_i and l_i^T are the right and left eigenvectors of A_c, respectively, corresponding to the eigenvalue β_i.

Theorem 1: If $WCE = 0$ and all rows of the matrix $H = WC$ are left eigenvectors of A_c corresponding to any eigenvalues, Eq. (9) holds true.

Proof: Let all rows of the matrix $H = WC$ be p left eigenvectors $(l_i^T,\ i = 1,2,\ldots,p)$ of A_c, i.e.,

$$H = [l_1 \quad l_2 \quad \cdots \quad l_p]^T \tag{11}$$

Then

$$Hv_i = 0 \qquad \text{for} \qquad i = p+1,\ldots,n$$

and

$$(sI - Ac)^{-1} = \frac{v_1 l_1^T}{s - \beta_1} + \cdots + \frac{v_p l_p^T}{s - \beta_p}$$

But we have that

$$WCE = HE = 0 \tag{12}$$

That is,

$$l_i^T E = 0 \quad \text{for} \quad i = 1,2,\ldots,p$$

Thus,

$$G_{rd}(s) = WC(sI - A_c)^{-1} E = 0$$

Hence, the residual is completely decoupled from the disturbances (unknown inputs), and robust fault detection is then achieved. The eigenstructure assignment approach for the design of the disturbance decoupling residual generator is as follows:

1) Compute the residual weighting matrix W so that $WCE = 0$.

2) Determine the eigenstructure of the observer: the eigenvalues of the observer are chosen according to the desired dynamic property of residuals. The rows of $H = WC$ must be p left eigenvectors of the observer; the remaining $(n-p)$ left eigenvectors will be chosen to ensure that the condition number of the modal matrix be reasonable.[20,21]

3) Compute the gain matrix K to assign the eigenstructure of the observer according to the required eigenstructure.[17,20-22]

It is clear that the observer feedback eigenstructure assignment problem can be handled by means of a transformation of the dual control form. On assignment of the right eigenvectors to the dual control problem, these eigenvectors become the left eigenvectors of the observer system.[20,21] The assignment of the controller right eigenvectors is a well-developed technique.[17,20-22] The assignability condition is that, for each eigenvalue β_i, the corresponding left eigenvector l_i^T must belong to the row subspace spanned by $\{C(\beta_i I - A)^{-1}\}$. If the left eigenvector assignability condition is not satisfied, we can consider an alternative method, i.e., to assign the right eigenvectors of the observer as columns of the matrix E.

Theorem 2: If $WCE = 0$ and all columns of the matrix E are right eigenvectors of A_c corresponding to any eigenvalues, then Eq. (9) holds true.

If the required observer eigenstructure is assignable (left or right), perfect decoupling can be achieved. On the other hand, if the required eigenstructure is not perfectly assignable, the eigenvectors must be chosen to be close, in a least-squares sense, to the desired eigenvectors. In this situation, the residuals also have low sensitivity to uncertainties due to approximate decoupling.

C. Design in the Discrete-Time Domain

Design in the discrete-time domain can be carried out in a similar way to the continuous-time domain case. However, some special properties exist in the discrete-time design. Here, we consider a dead-beat design, for which the derivation of the disturbance decoupling principle is very simple. Consider the discrete-time system described by

$$x(k+1) = Fx(k) + Gu(k) + Ed(k) \tag{13}$$

$$y(k) = Cx(k) + Du(k) + f(k) \tag{14}$$

The observer and residual equations are

$$\hat{x}(k+1) = (F - KC)\hat{x}(k) + (G - KD)u(k) + Ky(k) \tag{15}$$

$$\hat{y}(k) = C\hat{x}(k) + Du(k) \tag{16}$$

$$r(k) = W\left[y(k) - \hat{y}(k)\right] \tag{17}$$

Assume that $F_c = F - KC$; the system estimation error and residual equations are

$$e(k+1) = F_c e(k) + Ed(k) - Kf(k) \tag{18}$$

$$r(k) = He(k) + Wf(k) \tag{19}$$

The z transform of $r(k)$ is

$$\begin{aligned} r(z) &= H(zI - F_c)^{-1}\left[Ed(z) - Kf(z)\right] + Wf(z) \\ &= z^{-1}H(I + F_c z^{-1} + \cdots)\left[Ed(z) - Kf(z)\right] + Wf(z) \end{aligned} \tag{20}$$

For this case, the sufficient decoupling conditions are

$$HF_c = 0 \tag{21}$$

$$HE = 0 \tag{22}$$

Choose H and K in such a way that the rows of H are the left eigenvectors of F_c corresponding to zero-valued eigenvalues; then Eq. (21) holds true. Equation (22) means that some left eigenvectors are orthogonal to the disturbance directions. The residual will have dead-beat transient performance due to the assignment of zero-valued eigenvalues. The dead-beat designs produce minimal time response, and this feature can be exploited to good use in the aim to provide a high sensitivity to soft (incipient) faults. When the disturbance decoupling conditions of Eqs. (21) and (22) hold true, the residual is

$$r(k) = Wf(k) - HKf(k-1) \tag{23}$$

The fault signal is transmitted directly into the residual; the residual then gives the approximate shape of the fault. From the residual generation point of view, the residual $r(k)$ is a vector function of inputs and outputs of the system. From observer Eqs. (15-17), we can derive the z transform of residual $r(k)$ as

$$\begin{aligned} r(z) = {} & \left[W - H(zI - F_c)^{-1}K\right] y(z) \\ & - \left[WD + H(zI - F_c)^{-1}G_1\right] u(z) \end{aligned} \tag{24}$$

where $G_1 = G - KD$. If Eq. (21) holds true, Eq. (24) is changed into

$$r(z) = \left[W - z^{-1}HK\right] y(z) - \left[WD + z^{-1}HG_1\right] u(z) \tag{25}$$

That is,

$$r(k) = Wy(k) - HKy(k-1) - WDu(k) - HG_1 u(k-1) \tag{26}$$

It can be seen that Eq. (26) is a first-order parity equation (parity relation).[23,24] Equation (26) can be implemented directly to generate the residuals for fault detection. In this situation, the observer is not necessary, and this has significance for real-time application aspects.

III. Jet Engine System Description

The gas turbine jet engine can be described essentially as a heat engine that uses atmospheric air as a working medium to generate propulsive thrust and mechanical power. The central unit of the mechanical arrangement is comprised of two main rotating parts, the compressor and the turbine, and one or more chambers. The control system has the function of coordinating the main burner fuel flow and the propelling exhaust nozzle. There are other control variables, such as inlet variable flaps and rear compressor variable vanes. Under normal operation the control lever selects a desired fuel flow rate, which, in turn, determines the engine speed. The fuel flow is proportional to the exhaust nozzle area. The jet engine illustrated in Fig. 2 has the measurement variables N_L, H_H, T_7, P_6, and T_{29}.

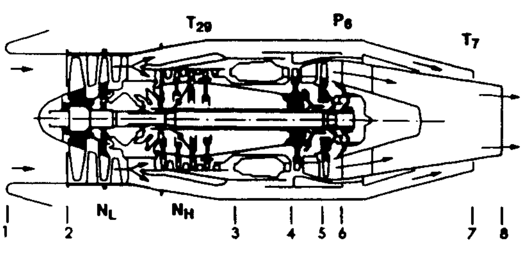

Fig. 2 Gas turbine jet engine.

N denotes a compressor shaft speed, P denotes a pressure, and T represents a measured temperature.

A thermodynamic simulation model of a gas turbine is utilized as a test bed for the evaluation of a fault detection scheme. The thermodynamic simulation model of the engine has 17 state variables; these include pressures, air and gas mass flow rates, shaft speeds, absolute temperatures, and static pressure. The system has two control inputs, the main engine fuel flow rate u_1 and the exhaust nozzle area u_2. This is a highly nonlinear dynamic structure that has grossly different steady-state operation over the entire range of spool speeds, flow rates, and nozzle areas. The linearized 17th model is used here for study. The nominal operating point is set at 70% of the demand high spool speed (N_H). For practical reasons and convenience of design, we choose to employ a fifth-order model to approximate the 17th-order model. Based on the fifth-order model, a fault detection scheme is designed. Moreover, on applying this scheme to a jet engine simulation for any other demand H_H set point, a mismatched case will result. The fault detection scheme is designed in the discrete-time domain. Using a sampling period of $\Delta T = 0.026$ s, the fifth-order discrete model matrices are

$$F = \begin{bmatrix} -0.9813 & 7.5320 & -0.5983 & 0.4857 & -0.6979 \\ 0.2838 & -0.0826 & 0.0779 & -0.0617 & 0.0928 \\ -6.8588 & 28.9161 & -2.0561 & 1.6083 & -2.2612 \\ 1.2235 & -5.6607 & 0.4020 & -0.3192 & 0.4141 \\ 13.2662 & -53.4047 & 4.7390 & -3.7710 & 5.3669 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.000139 & 0.000195 \\ 0.000067 & -0.000005 \\ 0.003188 & 0.000601 \\ 0.007840 & -0.000273 \\ 0.003123 & -0.001516 \end{bmatrix} \qquad C = I_{5\times 5} \qquad D = 0_{5\times 2}$$

As a consequence of using a reduced-order model to approximate the full-order dynamic system, modeling errors always exist. Hence, consideration of modeling errors takes on importance. Another problem is that the operating point of the system is varied according to realistic running conditions. In general, different operating points correspond to different plant models. In the practical situation, the design of the fault detection scheme is based on a fixed model. When the operating point is changed, a mismatch occurs; this is also considered as a modeling error in this paper. Discrete-time Eqs. (13) and (14) are used to represent the model. The uncertainty term $Ed(k)$ is used to represent the modeling error, with $d(k)$ as a disturbance vector and E as the disturbance distribution matrix. In the next section, we outline a method for estimating columns of the matrix E.

IV. Estimation of Disturbance Distribution Matrix

For all disturbance decoupling approaches, an important assumption is that the disturbance distribution matrix must be known, but a generalized approach to obtain the matrix is still lacking. Here, we attempt to estimate the matrix E.

Consider the system model Eqs. (13) and (14). To begin with, the matrix E is unknown. By defining $Ed(k) = d_1(k)$, we can generate estimations of all elements of the vector $d_1(k)$. In this way, it is possible to obtain some information about the matrix E. First, we assume that $d_1(k)$ is a slow time-varying vector, so that the system can be written in augmented form as

$$\begin{bmatrix} x(k+1) \\ d_1(k+1) \end{bmatrix} = \begin{bmatrix} F & I \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ d_1(k) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u(k) \tag{27}$$

$$y(k) = [C \quad 0]\begin{bmatrix} x(k) \\ d_1(k) \end{bmatrix} + Du(k) \tag{28}$$

An observer based on this model is used to estimate the disturbance vector. The input and output data $\{u(k), y(k)\}$ are derived from the original 17th-order continuous-time system. When the control input is $u = [1 \quad 1]^T$, the estimates of the elements of $d_1(k)$ are as shown in Fig. 3. From the diagram, it can be seen that the estimation of $d_1(k)$ becomes constant after a short transient. Our interest here is in the direction (distribution) of the disturbance, i.e., the relative magnitudes of all elements of the disturbance vector. It also can be seen that the relative magnitude of all elements of the disturbance vector converge to constant values. It can be further assumed that

$$d_1(k) \approx E_1 d_2(k) \tag{29}$$

Here, E_1 is a 5×1 vector, which is used to represent the direction of the disturbance, and $d_2(k)$ is a scalar that is the magnitude of the disturbance. E_1 itself represents the direction of $d_1(k)$. In fact, all directions of $\hat{d}_1(k)$ $(k = 0, 1, 2, \ldots)$ are slightly different. An optimal direction vector E_1^* that is used to approximate all directions of $\hat{d}_1(k)$ $(k = 0, 1, 2, \ldots)$ must be aligned to all of the directions of $\hat{d}_1(k)$ $(k = 0, 1, 2, \ldots)$ "as closely as possible." To obtain a reliable direction, we choose the steady-state estimates $\{\hat{d}_1(200), d_1(201), \ldots, \hat{d}_1(252)\}$ to compute this optimal direction. Here, an optimization problem can be quoted as

$$Q = \left[\hat{d}_1(200), \hat{d}_1(201), \ldots, \hat{d}_1(252)\right] \tag{30}$$

Normally, Q is a rank 5 matrix, although all columns of Q are nearly parallel. Hence, what would seem to make sense in this case is to choose a rank 1 matrix Q_0 that is as close as possible to the matrix Q, to choose a matrix Q_0, which minimizes

$$J = \|Q - Q_0\|_F^2 \tag{31}$$

subject to the constraint that rank $(Q_0) = 1$. Then E_1^* is a basis for the range space of Q_0. The optimization problem just posed is straightforward to solve. In particular, let the singular value decomposition of Q be given by

$$Q = U\Sigma V^T \tag{32}$$

where

$$\Sigma = \begin{bmatrix} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \sigma_3 & & 0 \\ & & & \sigma_4 & \\ & & & & \sigma_5 \end{bmatrix} \tag{33}$$

and U and V are orthogonal matrices. Here $\sigma_1 \le \sigma_2 \le \sigma_3 \le \sigma_4 \le \sigma_5$ are the singular values of E, ordered by magnitude. As shown by Lou et al.,[24] the rank 1 matrix Q_0 that minimizes Eq. (31) is given by

$$Q_0 = U\begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & 0 \\ & & & 0 & \\ & & & & \sigma_5 \end{bmatrix} V^T \tag{34}$$

Using the simulation data, we obtain

$$E_1^* = [0.4126 \quad -0.0617 \quad 1.5659 \quad -0.2776 \quad -2.9231]^T$$

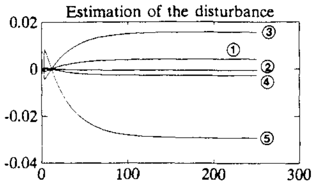

Fig. 3 Estimation of the disturbance vector for the step input case.

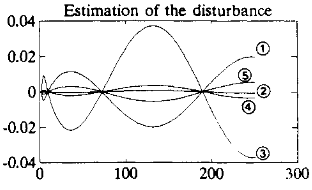

Fig. 4 Estimation of the disturbance vector for the sinusoidal input case.

Normally, the estimation of the disturbance vector $d_1(k)$ will be different for different inputs to the system. To check the generality of the direction of disturbance using the simulation, we change the input of the system to $u = [\sin(\pi t/3), \cos(\pi t/3)]^T$. The estimation of the disturbance signals is shown in Fig. 4. Although the magnitude of $d_1(k)$ is time varying, its direction (the relative magnitude of all elements) is almost constant. Following the procedure described previously, we obtain the approximate direction as

$$E_2^* = [0.5334 \quad -0.0768 \quad 1.9658 \quad -0.3698 \quad -3.7068]^T$$

In general, for a complex nonlinear system, the operating point will change according to the inputs and outputs of the process. Hence, it is instructive to consider the system to function at another operating point. In this study, this has been chosen as 95% N_H (or almost full dry power), using the nonlinear thermodynamic engine model to generate the linearized parameters. For this case of changed operation, the direction of the disturbances will also change. If we consider step inputs applied to both u_1 and u_2, we obtain the approximate direction as

$$E_3^* = [1.0511 \quad -0.1545 \quad 4.3087 \quad -0.9646 \quad -7.8283]^T$$

For a sinusoidal input, the approximate direction is

$$E_4^* = [1.1580 \quad -0.1644 \quad 4.3874 \quad -0.8722 \quad -8.2010]^T$$

Although there are differences between E_1^*, E_2^*, E_3^*, and E_4^*, the misalignment angles between them are very small. In fact, the misalignment angles are: $\angle(E_1^*, E_2^*) = 0.3764$ deg, $\angle(E_1^*, E_3^*) = 1.5633$ deg, and $\angle(E_1^*, E_4^*) = 0.5712$ deg. Thus, it is reasonable to say that the disturbance direction is almost constant (E_1^* is used as a representative in the study) for the system studied here, although the system is a fully nonlinear gas turbine model. The result is an interesting basis for further study.

V. Design of the Robust Fault Detection Scheme

A fifth-order discrete-time observer is used to generate the disturbance decoupling residuals. The first step for disturbance decoupling (robust residual generation) design is to compute the residual weighting matrix W such that $WCE = 0$ [Eqs. (12) or (21)] holds true. We choose

$$W = \begin{bmatrix} -0.3673 & -0.4413 & 0.6556 & 0.4095 & 0.2698 \\ -0.1159 & 0.8945 & 0.3343 & 0.2454 & 0.1205 \\ 0.8795 & -0.0501 & 0.3504 & -0.0343 & 0.3162 \end{bmatrix}$$

which ensure that $WCE_1^* = 0$, $WCE_3^* = 0$, $WCE_2^* \approx 0$, and $WCE_4^* \approx 0$.

The desired eigenvalues of the observer are $\{0, 0, 0, 0, 0\}$ such that the observer has a state dead-beat structure. The desired left eigenvectors of the observer are the rows of the matrix $H = WC = W$. The gain matrix of the observer can be derived using eigenstructure assignment. In this example, as all eigenvalues of the observer are zero and the C matrix is an identity matrix, the gain matrix is simply derived as $K = F$. Because $WCE_1^* = 0$, $WCE_3^* = 0$, $WCE_2^* \approx 0$, and $WCE_4^* \approx 0$ and the rows of the matrix $WC = H$ are the left eigenvectors of the observer corresponding to zero-valued eigenvalues, i.e., robustness conditions of Eqs. (21) and (22) hold true, the fault detection scheme is then always robust (such that disturbance decoupling always holds) when the system works at different operating points and different types of inputs.

VI. Performance Assessment of the Fault Detection Scheme

The robust fault detection scheme is applied to the jet engine system to detect the faulty sensors. Simulation is based on the 17th-order thermodynamic jet engine continuous-time model under open-loop control. A particular emphasis of this assessment study is the power of the method to detect soft or incipient faults that are otherwise unnoticeable in the measurement signals. These attributes are well illustrated in the following graphical time response results. As the fault detection scheme has been made robust against modeling errors, the scheme is able to detect incipient faults under conditions of modeling uncertainty. The uncertainty of the jet system model has been increased further by simulating the effect of random noise generated through a small malfunction in the fuel flow regulator system—to emulate the possibility of a high interference level arising in the electronic system. This has been achieved by adding a zero-mean Gaussian random signal, with a variance of 1% of demanded fuel flow, to the fuel-flow actuation signal in the model. The inputs to the system are $u = [1, \quad 1]^T$, and the initial values are zero. The faults are generated from a software fault simulator and superimposed on the sensor output signals. The linear model used has been based on a per-unit scaling of the engine dynamics and, hence, the final results have been scaled to give meaningful magnitudes.

Figure 5 shows the residual norm and the output estimate error norm for both fault-free and faulty cases. The result shows that the residual is very small in the fault-free case, i.e., disturbance decoupling is achieved. The output estimation error is very large even when no faults occur, and this cannot be used to detect the fault reliably. Figure 6 shows the faulty output of the pressure sensor P_6; the fault is very small compared with the output, and, consequently, the fault cannot directly be detected in the output. The corresponding residual and the output estimate error for this faulty case are shown in

a)

b)

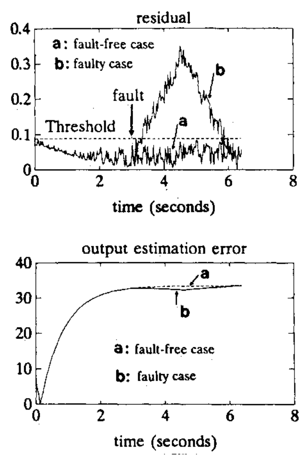

Fig. 5 Residual [$\underline{r}(k)$] norm and the output estimation error [$\underline{e}_y(k)$] norm.

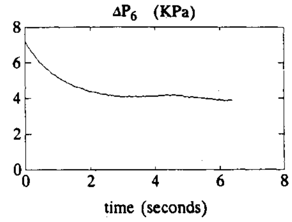

Fig. 6 Faulty output of the pressure sensor P_6.

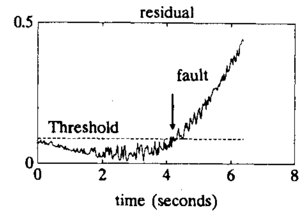

Fig. 7 Residual norm for the case a parabolic fault occurs on the spool speed sensor N_H.

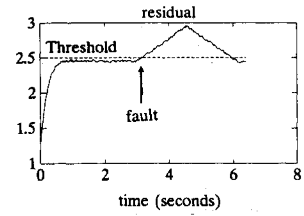

Fig. 8 Residual when a fault occurs in the temperature sensor T_7 and the jet engine system works at another operating point (95% N_H).

Fig. 5. It can be seen that the residual has a very significant increase when the fault occurs. Despite the actuation noise, a threshold can easily be placed on the residual signal to declare the occurrence of faults. But one cannot be sure whether a fault has even occurred in the system when using the information from the output estimate error. Figure 7 shows the fault detection performance of the residual for detecting a parabolic fault in spool speed sensor N_H. The results demonstrate the efficiency of the robust model-based approach in the role of robust fault detection.

In general, for a complex nonlinear system the operating point will change according to the inputs and outputs of the process. Hence, it is instructive to consider the system functioning at different operating points. A robust fault detection scheme should work well for a range of operating points. To assess the robustness performance, the scheme is used to detect the fault when the system is working at another operating point (in the presence of demanded changes in high compressor speed N_H); the result is shown in Fig. 8. Note that, although the magnitude of the residual has been changed, the fault can also be easily detected from the significant increase of the residual.

VII. Concluding Remarks

One critical limitation of the analytical redundancy approach to fault detection is a consequence of the fact that uncertainties acting upon the observer are inevitable. In the jet engine system, the effects of uncertainty are more pronounced compared with other systems. This paper has focused on a robust fault detection method using eigenstructure assignment technique for the design of a disturbance decoupling (robust) residual generator. Some left (or right) eigenvectors of the observer are assigned to be orthogonal (or parallel) to the disturbance directions. Thus, disturbance decoupling (robustness) is achieved. The robustness aspects have made it possible to place a low threshold for the detection of soft faults. The special properties of discrete-time design are also emphasized in this paper. A discrete robust observer has an output dead-beat structure. It yields a response with minimum time transient.

Designs have been described to illustrate the application of the method to a typical gas turbine engine sensor system. The fault detection scheme is designed on the basis of a reduced-order model of the 17th-order linear system obtained from a nonlinear gas turbine engine simulation package. The system offers a big challenge to fault detection due to the inevitable presence of modeling errors. The important issues are demonstrated clearly using a discrete-time design based on the dead-beat principle. Excellent results have been obtained, and these indicate the effectiveness of the method for detecting soft (small) faults. Moreover, the fault detection scheme is robust with respect to variations of the operating point. Discussions with other researchers working in this field indicate that the approach to robust decoupling considered may be considerably more efficient than the alternative method of estimating all of the parameters of the system state-space matrices. The estimation of the E matrix can be considered as a parameterization of the unknown elements of the system matrices, and the approach can thus be more efficient.

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